RAK-003-1014008 Seat No. ________

B. Sc. (Sem. IV) (CBCS) (W.E.F. 2016) Examination

March / April - 2019

MATH-04(A) : Mathematics

(Linear Algebra & Differential Geometry Theory)

[New Course]

Faculty Code : 003
Subject Code : 1014008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are compulsory.

1 (a) Answer the following in brief : **4**

(1) Define Linear dependence.

(2) Define Improper subspace.

(3) Define Polynomial space $P_n(R)$.

(4) If $V = R^2$, for $(x,y),(w,z) \in V$ and $\alpha \in R (x,y)+(w,z)=(x+w,y+z)$ and $\alpha(x,y)=(\alpha x,y)$, if $\alpha = 2$, $\beta = -2$ and $(x,y)=(1,4)$, then find the value of $(\alpha+\beta)(x,y)$.

(b) Answer any one question : **2**

(1) Check whether V is vector space, where $V=\{(x,y): x,y \in R\}$ for $(x_1,y_1),(x_2,y_2) \in V$ $(x_1,y_1)+(x_2,y_2)=(x_1,y_1)$ and for $\alpha \in R$ $\alpha(x_1,y_1)=(\alpha x_1, \alpha y_1)$.

(2) Check whether the sub sets $\{(1,1,-1),(1,0,1),(1,1,0)\}$ of vector space R^3 and L.D. or L.I.

(c) Answer any one question : **3**

(1) Prove that intersection of any two subspace of vector space is also a subspace.

(2) Check whether the set $W=\{(x,y,z): x+y+z=0,\ x,y,z\in R\}$ is subspace of R^3.

(d) Answer any one question : **5**

(1) If a vector $\overline{v_k}\,(1\le k\le n)$ of set $\{\overline{v_1},\overline{v_2},..........,\overline{v_n}\}$ is a linear combination of remaining vectors $\overline{v_1},\overline{v_2},........\overline{v_{k-1}},\overline{v_{k+1}}.......,\overline{v_n}$, then prove that SP $\{\overline{v_1},\overline{v_2},..........,\overline{v_n}\}$=SP $(\overline{v_1},\overline{v_2},........\overline{v_{k-1}},\overline{v_{k+1}}.......,\overline{v_n})$.

(2) Check whether V is vector space, where $V=\{(x,y): x,y\in R\ x>0,\ y>0\}$ for $(x_1,y_{1,}),(x_2,y_2)\in V$ $(x_1,y_1)+(x_2,y_2)=(x_1x_2,\ y_1y_2)$ and for $\alpha\in R$ $\alpha(x_1,y_1)=(x_1^{\alpha},\ y_1^{\alpha})$.

2 (a) Answer the following in brief : **4**

(1) Define Dimension.

(2) Write the standard base of $M_2(R)$.

(3) Define Base.

(4) What is the dimension of $P_n(R)$?

(b) Answer any one question : **2**

(1) If W is subspace of finite dimensional vector space V. Show that $\dim W \le \dim V$.

(2) Check whether the subset

$A = \{(0,1,2,1),(1,2,-1,1),(2,-3,1,0),(4,-2,-7,-5)\}$ of

R^4 are basis of R^4.

(c) Answer any one question : **3**

(1) If $\{e_1, e_2, e_3\}$ is standard base of R^3 then show that

$\{e_1 + e_2, e_2 + e_3, e_3 + e_1\}$ is also base of R^3.

(2) Prove that $\{1-x, 1+x, 1-x^2\}$ is base of $P_2(R)$.

(d) Answer any one question : **5**

(1) If $W_1 = \{(x_1 + x_3 - x_4) \backslash x_1 + x_3 - x_4 = 0\}$ and

$W_2 = \{(x_1, x_2, x_3, x_4) \backslash x_1 + 2x_2 = 0\}$ are subspace of R^4.

Find $\dim W_1, \dim W_2, \dim(W_1 \cap W_2)$ and show that

$W_1 + W_2 = R^4$.

(2) Prove that the set $A = \{(1,2,1),(2,1,0),(1,-1,2)\}$ forms a basis for R^3. Find coordinate $(1,1,-1)$ with respect to this base.

3 (a) Answer the following in brief : **4**

(1) Define Zero linear transformation.

(2) Define Linear function.

(3) Define Nilpotent linear transformation.

(4) Define Kernel of a linear transformation.

(b) Answer any one question : **2**

(1) Find linear transformation $T : R^3 \to R^3$ such that $R(T) = SP\{(1,0,1), (1,2,3)\}$.

(2) Find N_T and $n(T)$ for the linear transformation $T : R^3 \to R^2$, $T(x, y, z) = (x - y + z,\ x + y - z)$, $\forall (x, y, z) \in R^3$.

(c) Answer any one question : **3**

(1) For linear transformation $T : R^3 \to R^3$, $T(a, b, c) = (a - b + c, b - c, c)$, $\forall (a, b, c) \in R^3$, find T^{-1} if exists.

(2) Prove that composition of two linear transformation is again a linear transformation.

(d) Answer any one question : **5**

(1) State and prove Rank-Nullity Theorem.

(2) Prove that $L(U,V)$ is a vector space over R with respect to addition and scalar multiplication of linear transformation.

4 (a) Answer the following in brief : **4**

(1) If $\dim U = 4, \dim V = 3$, then find the $\dim L(U,V)$.

(2) Define Eigen value of a linear transformation.

(3) Define Dual of vector space.

(4) Define Adjoint of linear transformation.

(b) Answer any one question : **2**

(1) $B_1 = \{1, x, x^2\}$ and $B_2 = \{1, x, x^2, x^3\}$ are bases of $P_2(R)$ and $P_3(R)$ respectively. Find the linear transformation $T : P_2(R) \to P_3(R)$ related to the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

(2) Let $T : R^2 \to R^2$, $T(x, y) = (x, -y)$, $\forall (x, y) \in R^2$ and $B_1 = \{(1,1), (1,0)\}$ and $B_2 = \{(2,3), (4,5)\}$. Then find $[T; B_1, B_2]$.

(c) Answer any one question : **3**

(1) A linear transformation $T : U \to V$ is defined by $T(u) = a.u, \forall u = (x_1, x_2, x_3) \in U$ and $a = (a_1, a_2, a_3)$ is constant vector. U and V have standard Euclidian bases find $[T; B_1, B_2]$.

(2) A linear transformation $T : P_2(R) \to P_3(R)$ is defined by $T(p(x)) = \int_0^x p(x)\,dx$. $B_1 = \{1, x, x^2\}$ and $B_2 = \{1, x, x^2, x^3\}$ are bases of $P_2(R)$ and $P_3(R)$ respectively find $[T; B_1, B_2]$.

(d) Answer any one question : **5**

(1) Find the Eigen value and Eigen vector for the linear transformation $T : R^3 \to R^3$,

$T(a,b,c) = (-2b - 2c, -2a - 3b - 2c, 3a + 6b + 5c)$,

$\forall (a,b,c) \in R^3$ by considering the standard basis of R^3.

(2) Let $T : V \to V$ be a linear transformation and let B be any basis of V. Then T is singular if and only if $\det([T; B]) = 0$.

5 (a) Answer the following in brief : **4**

(1) Define Double point.

(2) Define Singular point.

(3) Define Point of inflexion.

(4) Find the radius of curvature of the curve $s = 4a \sin \psi$.

(b) Answer any one question : **2**

(1) Find the radius of curvature of the curve $s = c \cosh \frac{x}{c}$.

(2) Prove that $y = \log x$ is convex upward everywhere.

(c) Answer any one question : **3**

(1) Find the all asymptotes of the curve

$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 = 1 \cdot$

(2) Find the radius of curvature at origin for the curve $x^3 + y^3 = 3axy$ using Newton's method.

(d) Answer any one question : **5**

(1) Show that radius of curvature of any point on the cardiod $r = a(1+\cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$ and prove that $\frac{\rho^2}{r}$ is constant.

(2) Find the position and nature of double points of the curve $x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$.
